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Mathematics News Letter

SUBSCRIPTION
50c.
Per Year

Published in the interest of the **Louisiana-Mississippi Section of the Mathematical Association of America** and the **Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics.**

A Challenge to Forward-looking Mathematics Teachers in the Colleges and High Schools of Louisiana and Mississippi.

Louisiana-Mississippi Section, Mathematical Association of America

S. T. SANDERS, *Chairman.* Baton Rouge, La.
P. K. SMITH, *Sec'y-Treas.,* Hattiesburg, Miss.
HAL FOX, *Vice-Chairman,* Mississippi A. & M.
J. A. HARDIN, *Vice-Chairman* Shreveport, La.

Louisiana-Mississippi Branch National Council of Teachers of Mathematics

W. C. ROATEN, *Chairman,* De-Ridder, La.
MISS IDA K. SMITH, *Vice-Chairman,* Meridian, Miss.
MRS. L. T. LONGMIRE, *Sec'y Treas.,* Alexandria, La.

Vol. 2 BATON ROUGE, LA., FEBRUARY, 1928 No. 4

LOUISIANA, MISSISSIPPI AND SCIENCE

In "*Science*", weekly organ of the American Association for the Advancement of Science, was recently published an interesting compilation of statistics entitled, "Registration at Nashville by States and Provinces", and was intended to show the "residence distribution" of those who attended the recent meeting of the American Association for the Advancement of Science, which was held in Nashville December 27-30. In the compilation, registrations of attendance from forty-seven States are given. The number of scientists in attendance from Louisiana is recorded as 43. This record is surpassed by the attendance records of only two other states of the south. These are Tennessee and North Carolina, attendance from North Carolina being 47, that of Tennessee (outside of Nashville) 76. This attendance figure for Louisiana exceeds the attendance figures of thirty-three other States of the forty-seven represented at Nashville, or, put otherwise, Louisiana was the 14th State from the top in the matter of numerical representation at this great scientific meeting. The attendance figure for Mississippi was also highly creditable, that figure being 29, or greater than the attendance record of twenty-one other States of the Union.

ABOUT THE JACKSON PROGRAM

The mathematical public of Louisiana and Mississippi should be keenly interested in the following announcements concerning the Jackson program. On March 30, 31 the joint program

of Section and Council will be featured by papers from President B. M. Walker, of Mississippi A. & M. College, Professor C. N. Wunder, of the University of Mississippi, Professor H. E. Buchanan, of Tulane University, Professor W. Paul Webber, of Louisiana State University, and Major Jas. P. Cole, of Louisiana Polytechnic Institute, head of the mathematics department of that institution. It is expected that we shall be able to publish the subjects of the papers to be contributed by these gentlemen in the March issue of the News Letter. The principal address of the meeting will be delivered on Friday evening, March 30, by Professor C. N. Moore, of the University of Cincinnati, who will be present as the representative of the Mathematical Association of America.

Chairman Roaten, of the Louisiana - Mississippi Council Branch reports that Miss Norma Touchstone, of Bolton High School, Alexandria, will contribute to the Council program. Other Council participants will be announced a little later.

VECTORS AND SCALARS OF MATHEMATICAL INTEREST

The creation of *measures* is one of the great achievements of mathematics. Scarcely anything in these times exists which has not been, or cannot be measured. Even abstract entities, such as our intelligence, our liability to disease, to death, or to marriage, the chance of our being run over by an automobile, or the degree of our immunity from a stale joke—all these are subject to more or less precise measurement, if this should seem desirable.

Then, there are those things—also the creation of mathematics—which, though not of a sort that can be measured, yet have a definite relation to measures. These things belong to the *vector* aspects of measures. The measurement of a quantity is often a laborious task, demanding repeated experiment, painful accuracy, and a trained intelligence. On the other hand, all passing observers can note the sense of a vector, its plus or minus direction, its additive or subtractive quality. No trained vision is required, only a casual glance, to tell if the ship sails north or south, if the mercury is rising or falling, if an army is retreating or moving forward.

Nor is it required that one know mathematics in order to pass judgment on the positive or negative or zero character of the professional interest of him, or her, whose daily work is to teach mathematics. We mathematicians are compassed about with many witnesses. It may take another mathematician to determine a numerical measure of my

interest in the task of developing a maximum service out of my mathematics, but a chance visitor to my class room is able to tell if my mathematical interest is a plus or minus vector. In a multitude of cases the measure-to-scale is incomparably less significant than its sense aspect. The widow's mite was ridiculously small, but its vector significance was great when she cast it, all she had, into the treasury. Possibly the mite of those times had the value of a half-dollar of today, but even so it is safe to assume that few teachers of mathematics in this generation are unable to invest fifty cents in a cause aimed to assist them to professional advancement. As a measure, the amount is small. Invested in our Mathematics News Letter it becomes a flying signal, a positive vector, of mathematical interest.

Negative vectors? The temptation to enumerate these will be put aside as bad psychology. Not even the number of our paid-up subscriptions shall here be printed. Each mathematician and mathematics teacher shall be left to take the census of his own minus signs, if he so chooses, while, from his own corner he surveys the forces which for nearly two years have been directed to lift mathematics to its proper level in school programs, and to raise to proper standard the profession of mathematics teaching in our Louisiana- Mississippi territory.

FRESHMAN MATHEMATICS

Chemistry, biology, and even certain branches of agriculture are today making demands on mathematics which fifty years ago were not made. The fact makes apparent the advisability of some departure from the traditional course in first year college mathematics.

If the present usual freshman mathematics course material were so modified as to furnish an increased amount of specific technical mathematics for engineering, science, statistical or industrial students, or for those about to major in some form of applied science, programs of mathematical discipline, or programs aiming for culture, or for training in logic, by way of mathematics, need not be affected in the slightest degree.

A chapter on statistical method does not have to be devoid of culture elements, if by culture is meant the assimilation of historical material. A few pages on the use of the slide rule need not be dissociated from a theoretical treatment of logarithms, or from a certain amount of pleasant review of the facts about the development of logarithms by such men as Napier and Briggs. Technical formulae adapted to determine weights, pressures, mo-

ments, rates, distances, error amounts, can be viewed in the function light as well as they could if they were not being handled primarily for their applications.

SHALL WE COOPERATE ?

By W. C. ROATEN
DeRidder High School

Whether well founded or not, there is a general feeling that there is a lack of sympathetic cooperation between the college teachers and those who work in the secondary schools. On one side it is charged that the college professor looks on the feeshman as an ignoramus, and that he takes a fiendish delight in cutting him down at the earliest possible opportunity; while ,on the other hand, there comes from another source the complaint that the secondary teacher "don't know nuthin' no how" and that his chief professional duty is to draw his munificent salary from an indulgent public. Whether any or all of these notions are true has nothing to do with the present situation. That there is need of a closer cooperation between the colleges on one hand and the secondary schools on the other is admitted by every one who has studied the conditions.

This spirit of cooperation must be based on the feeling that the colleges and the secondary schools are working with the same materials with the same general end in view, and that mutual confidence and respect on the part of all teachers will conduce to the realization of this general aim.

There has been instituted within the States of Louisiana and Mississippi an organization which has for its fundamental aim a thought which if developed will go far toward bringing these two teaching forces into closer cooperation. When the Louisiana-Mississippi Section of the Mathematics Association of America and the Louisiana - Mississippi Branch of the National Council of Teachers of Mathematics united forces at the Shreveport meeting last March, the idea uppermost in the minds of the representatives of the two organizations was to unite as closely as possible the college and the secondary mathematics teachers. If this aim is to to be realized even approximately, these teachers must rally to the support of the leaders.

For this purpose are they being urged to give their support to the News Letter and to attend the meeting of the two organizations at Jackson, Miss., March 30-31.

THE APPROACH TO PLANE GEOMETRY

By LEORA BLAIR
Louisiana State Normal College

More and more teachers are coming to realize that pupil success in plane geometry is largely dependent on the right kind of a start. This is due partly to the values that lie in conserving the interest inherent in all totally new work, and partly to the cumulative character of the subject itself. One cannot build the structure of geometry without a foundation.

While the foundation for geometrical work usually consists of definitions, axioms and postulates, we are finding out that better results may be obtained if the approach is pedagogical rather than logical. This means that at the outset the pupil is given only a minimum of definitions and principles, and that great effort is made to find material and work that he can really master. Here are a few suggestions:

1. Let the first proofs—the equality or congruence of triangles—be experiemental in nature. Paper cutting and placing enable the pupil to understand the real meaning of congruence. He does not know what a rigorous proof is, so he is completely satisfied. Any teacher of experience knows how difficult it is to try to prove something to a pupil who does not realize that a proof is needed. It is of greater importance that the subject seem logical than that it be logical in the strict sense. After a few weeks when the idea of a logical proof is developed, these early theorems should be proved. They will then be readily understood.

2. To develop the idea of a geometric proof, give many simple original exercises. These are the soul of geometry. They should be carefully graded and should be so simple that every pupil in the class can do a majority of them. Any good text will provide some of these exercises, but most texts do not contain enough. The teacher should prepare a note-book for himself with suitable supplemental material if he does not possess two or three other good textbooks. The class period at first can be devoted to group thinking. Advances in difficulty can be easily made in this way. As a part of the assignment a new theorem may be presented as an exercise. The pupil is not tempted to memorize it, because he can develop the proof. If memory work is needed, it is valuable because it is based on material already comprehended.

3. Construction work also affords great opportunity for pupil participation in the early part of

the course. Not much time can be devoted to drawing fancy figures, but simple construction work such as bisecting an angle or drawing a triangle with its sides equal to three given lines can be taught as a means of creating more original exercises. It may properly provide many numerical exercises which are very useful. It may also be used in some simple outdoor work which, if given occasionally, will help to motivate geometry.

Such an approach requires very active work on the part of the teacher. Enthusiasm and a pleasing personality yield a great reward. With this method pupils will not only learn geometry, but how to study geometry, which is perhaps of greater value. The class period will be really welcomed by most of the class because it provides work with the spirit of a contest in which all may succeed.

THE GROWTH OF MODERN METHODS OF COMPUTATION

By H. E. BUCHANAN
Tulane University

There are five great steps which have led to our present powerful methods of computation. They are:

1. The idea and symbols for integers and common fractions.
2. The invention of the zero and its use in place value.
3. The invention of decimal fractions.
4. The invention of logarithms.
5. The development of computing machines.

We consider these briefly.

The art of counting is prehistoric. Man found it necessary to be able to count small numbers probably 5 to 10 thousand years ago in order to keep up with the number of his children, or the number of arrows that a deer was worth. It is clear from the remnants of ancient languages that still exist that he first counted on the fingers of one hand. Witness: the constant recurring V in the Roman notation. Indeed most of the American Indians used the quinary system at the time the white man found them, the Mayas being a conspicuous exception. They used 20 as a base and had a zero with place value. Later, man counted on the fingers of both hands. This proved a convenient unit and it is from this that we derived our decimal notation. Some tribes, as the need for it arose, counted all their fingers and their toes. I have read that one tribe,

perhaps in Patagonia, had a word for the number 21 which when translated into our language meant "one on the other man." But the number 20 as a base proved too cumbersome, and at the dawn of history we find all the people of the Mediterranean littoral united on 10 as a base of the number system. It would have been better if man had sprouted another finger on his hand for all the simple fractions that we use so much, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$ could be expressed as simple duo decimal fractions thus: .6 .4 .3 .2 instead of .5 .333. . . .25 and .1666. . .

Charles the XII of Sweden, at the time of his death, was planning to force his people to use the duodecimal system. He did not carry out his plans but even the power of a King is not sufficient, probably, to force a nation to use a number system it does not want to use. We are too firmly intrenched in the use of the decimal system ever to change. Even the French Revolution, which swept away almost everything else left the decimal system more firmly intrenched.

The next step in the development of arithmetic was the common fractions. That came as a result of increasing trade relations, that is, as a result of the pressing needs of commerce. This step was taken by the Babylonians. They introduced the sexagesimal fractions, that is, fractions with denominator 60. For example, they wrote $\frac{1}{2}$ as 30/60 and all the other fractions that they used were reduced to that denominator. It seems very strange to the novice that such a large denominator should be used and that the Babylonians were apparently unable to think with fractions till they had reduced them to this denominator. There are two remarks we may make which will partially, at least, clarify this:

First. The idea of fractional relations is not an easy one and the easiest way of doing things is seldom found on the first attempt.

Second. The Babylonians had already developed a fair calendar, and a system of measuring time. Our present system of years, months, days, hours and minutes is due to them. Every time you look at your watch you unconsciously do homage to the astronomers who lived on the banks of the Euphrates 4000 years ago. Their system of dividing time into parts doubtless influenced their division of integers into parts. Cantor suggests that their year of 360 days was divided into 6ths because they knew that the radius of a circle will step around the circumference as a chord exactly 6 times.

He cites as proof of this fact that all their chariot wheels had six spokes. They were able to do all the operation necessary for their commerce which was quite extensive. There was one firm,

Egebi & Sons, which did a banking business for 1500 years, through half a dozen different Kingdoms.

We transfer our story now to Egypt whose people developed a civilization as early as the Babylonians. There we find all education in the hands of the priesthood. An Egyptian priest, Athmes, wrote a book on mathematics about 1700 B. C. (A papyrus found and translated by Eisenlohr, 1871). In it are a few problems on algebra with directions for solving; a few rules on geometry stated empirically and some of them incorrectly; and a large body of work on reducing fractions to unit fractions. The Egyptian minds could not comprehend a fraction unless its numerator was unity. This book was reputed to be a copy, with probably some additions, of an older book dating back to 3000 B. C. Very careful directions are given on how to break up any fraction into the sum of unit fractions and many examples are worked out. Thus he wrote $\frac{1}{6}+1/18$ for $2/9$ and gave a table faintly resembling our tables of logarithms for breaking up any fraction. By this table he could obtain results like $2/35= 1/42+1/86+ 1/301$. Mirabile Dictu! How such a table was constructed is puzzling. Probably it was accumulated through hundreds of years of effort on the part of many priests.

The beginnings of geometry are in Egypt also. In fact the Egyptian word for geometry means literally, rope stretcher, suggesting that they stretched ropes to re-locate land lines which were undoubtedly obliterated each year by the overflow of the Nile. The Greeks went to Egypt to study and the early Greek geometers, Thales, for example, frankly admit their debt to Egypt. But the Greeks soon outstripped their Egyptian teachers.

Other papyri have been found. One, presumably written 500 A.D. shows no improvement throughout a period of 2000 years. Possibly this was due to the fact that mathematics, being in the hands of the priesthood, became sacred, therefore unchangeable. At least there is room for thought in the fact.

The Romans contributed very little to the development of arithmetic. In geometry they were not even good imitators. The two things they have left us in arithmetic are (1) the Roman notation which we use now on watches and clocks and as chapter headings in books, and (2) the duodecimal fractions. Neither of these is of any decided value to us; but they are interesting curiosities. Their notation, as mentioned before, shows unmistakable evidence of having grown up from finger counting in the V. It has a subtractive principle which does not occur in any other system of

notation. For example IV and IX, for four and nine respectively. Their fractions are interesting because they have the denominator 12 or a multiple of 12. Any fraction that could not be represented as 12ths, 24ths, or 48ths or 72nds, was taboo to them and must be approximated by some duodecimal fraction, indicating again the long struggle the race had to conquer the difficulties encountered in handling fractions—a difficulty met, but not conquered, by Babylonians, Egyptians, Greek and Roman. It is only in the last 300 years that we have obtained complete mastery of them. Perhaps we ought to be more patient with students who find fractions difficult since the race itself had such a struggle with their mastery!

We have, so far, briefly outlined the advances in the development of integers and fractions made by three different nations. Each carried his own notation to great lengths but reached a limit of progress in the cumbersomeness of the notation itself. It fell to the lot of the Hindus to furnish us the great idea which revolutionized arithmetic computations, paved the way for the introduction of decimal fractions, logarithms and the computing machines. I refer to the invention of the zero and its use in the principle of place value. It is undoubtedly one of the noblest conceptions of the human mind and came out of a nation whose people we, wrongly I suspect, regard as our inferiors socially and intellectually.

The zero and its use, was in the literature of the Hindus in 800 A. D. It is very likely that they had known its use more or less perfectly since 300 A. D., that is, for 500 years. Gradually its use, following trade routes, became known to the Arabs and was adopted by them together with the symbols 1, 2, - - - 9 for the first 9 integers. Leonardo of Pisa, a merchant and banker in 1200 A. D., brought this notation to Italy and changed the entire system of bookkeeping in his counting houses so as to conform. From there it followed trade lines to Germany, France, Spain and England. By 1500 A. D., it was firmly intrenched in all of Europe, but the use of the Roman notation did not die out easily. As late as 1821 an arithmetic was published in Germany using solely the Roman notation and trying to revive its usage in the schools.

So far, we have used an ascending scale of decimals, 1, 10, 100, 1000 and fractions in between. It was a long time before the human race grasped the descending scale,

$$1, \frac{1}{10}, \frac{1}{100}, \frac{1}{1000}.$$

The invention of the notation for decimal fractions is due to several men, particularly to Beyer,

Birgi and Stevin. The last one, Simon Stevin, a Belgian of Bruges, is usually given the credit, for he wrote a little book entitled "*La Disme*", the tenths, in 1585, in which he set forth their properties, gave many examples of their usefulness, but he did not introduce the decimal point. Instead he put a zero over his decimal parts thus: $16.21 = 16\overset{0}{2}\overset{0}{1}$. That several men were concerned in this development is not unusual. With one exception no great step has ever been made in mathematics by the isolated study of one man. Advances are made piece-meal by many men and the parts are put together in print probably by a man who has not helped in the discoveries. Euclid, for example.

The invention of logarithms is the one exception. This was accomplished solely by Baron Napier, a Scotchman, and given to the world in 1614. In it he introduced the decimal point and used the decimal fraction of Stevin throughout.

His method is very difficult to understand. I shall content myself by constructing a small table of logarithms and illustrating their use. Take the two sequences of numbers,

0	1	2	3	4	5	6	—
1	2	4	8	16	32	64	—

The product of any two numbers in the lower sequence can be found by adding the corresponding numbers in the upper sequence, running along the upper sequence to the sum, underneath which sum we find the product. For example, the product 4x 8 may be found by adding the 2 and 3 which are above 4 and 8, underneath 5 we find the product 32. Similarly, the quotients of any two numbers in the lower sequence may be found by subtracting the corresponding numbers above, and underneath the difference the quotient is found. One can equally as well raise any number in the lower sequence to any power or extract any root. One example will suffice to illustrate this: To find $\sqrt[3]{64}$ divide 6 by 3, under 2 is 4 which is the required root.

The two sequences above enable us to do multiplication, divisions, raising to powers and extracting roots. The difficulty in practice is that there are not enough numbers in the lower sequence. This can be remedied by noticing that we can thicken the lower sequence by placing in between each pair of numbers the square root of their product and in the upper sequence the corresponding number is half the sum. For example, between 4 and 8 we place $\sqrt{32} = 5.65$ - - - and above it 2.5. By continuing this process we may make the lower sequence as thick as desired so that any number whatever may be found in it. These two sequences constitute a table of

logarithms. The numbers in the upper sequence are the logarithms of the numbers in the lower. Since the lower numbers are powers of 2 we say that this is a table with base 2. This is essentially what Napier did. It took him twenty years to compute his table. After its publication, Briggs, a professor in Cambridge University, visited Napier and suggested that the tables would have been more useful if they had been computed with base 10. Napier saw the advantage of this at once. It had taken him twenty years to compute his first table. Without a moment's hesitation he began to recompute the entire table! This is one of the finest examples of what Andy Gump would call "intestinal fortitude" in the history of science. No exploit of the football field can equal it.

In a short article such as this we cannot discuss the computing machines. They vary all the way from a two-dollar adding machine to machines which will multiply and divide accuiately to many decimal places. There is no excuse for error now in computations of a standard type, but the computing machines do not do away with the need for accuracy in the student, for much computation cannot be reduced to the standard types.

GENERALIZATIONS OF EUCLID

By I. C. NICHOLS
Louisiana State University

What is the relation of high school mathematics to college mathematics? There are several worthwhile replies to this question. We wish to give only one of them, and then elaborate briefly: *High school mathematics treats more of the particular; college mathematics treats more of the general.*

It is indeed a distinct gain to acquire a knowledge of a general proposition whereby several other propositions may be made corollaries under it. Such knowledge gives a better perspective of the subject at hand, and certainly it makes for an economy of time and energy. Let us illustrate. Take the proposition of concurrent lines. On page 78 of our adopted text in high school geometry, we have it proven that the medians of a triangle are concurrent, that the bisectors of the internal angles of a triangle are concurrent, and that the perpendiculars from the vertices of a triangle on its sides opposite are concurrent. All three of these propositions are but corollaries of the converse of Ceva's Theorem; the proof of which is no more

beyond thehigh school student than are many of the theorems actually included in his course. While space does not permit this proof here, still we shall give the proof of the direct theorem and then, with this proof before them, those so desiring may prove the converse theorem by the familiar method of *reductio ad absurdun.*

I. *Ceva's Theorem;** *The lines joining the vertices of a given triangle to a given point determine on the sides of the triangle six segments such that the product of three of these segments having no common end is equal to the product of the remaining three segments.*

Proof: Let PA, PB, PC be the lines joining the vertices of the given triangle ABC to the given point P. Let L, M, N, be the traces of these respective lines on the sides of the triangle ABC. Draw BD and CE parallel to AL, and meeting CN and BP produced in the respective points D and E.

Then from the similar triangles BLP and BCE, we have

$$\frac{BL}{BC} = \frac{LP}{CE} \text{ (1).}$$

And from the similar triangles CBD and CLP, we have

$$\frac{BC}{LC} = \frac{BD}{LP} \text{ (2).}$$

Hence from (1) and (2), by multiplication, $\dfrac{BL}{LC} = \dfrac{BD}{CE}$ (3).

Again from the similar triangles CEM and APM, we have

$$\frac{CM}{MA} = \frac{CE}{AP}, \text{ (4),}$$

and from similar triangles BDN and APN, we have

$$\frac{AN}{NB} = \frac{AP}{BD}, \text{ (5).}$$

Hence from (3), (4), and (5), by multiplication, we have

$$\frac{BL}{CL} \cdot \frac{CM}{MA} \cdot \frac{AN}{NB} = \frac{BD}{CE} \cdot \frac{CE}{AP} \cdot \frac{AP}{BD} = 1.$$

Hence BL . CM . AN = CL . MA . NB, which was to be proven. Hence the theorem above.

II. *The converse Theorem is: If three points taken on the three sides of a triangle divide these sides into six segments such that the product of the three segments having no common end is equal to the product*

*Proper acknowledgments are made to Court's College Geometry.

of the three remaining segments, then the three lines joining the three points to the opposite vertices of the triangle are concurrent.

Without giving the proof of this theorem for the reason just stated above, let us examine in connection with it the three propositions on concurrent lines given respectively as exercises 5, 2 and 4, page 78, of Wentworth-Smith Plane Geometry:

(a) *The medians of a triangle are concurrent:*

If L, M, N are the mid-points of the sides of the triangle, then we shall have AN = NB, BL = LC, CM = MA. Hence, by multiplication, AN . BL . CM = NB . LC . MA. Obviously the conditions of our Theorem II are fulfilled, and hence the medians AL, BM, and CN are concurrent.

(b) *The bisectors of the internal angles of a triangle are concurrent:*

Again, if AL, BM, and CN are the bisectors of the respective angles A, B, and C of the given triangle ABC (since the bisectors of the angles of a triangle divide the respective sides opposite into segments proportional to the sides about the angle bisected) we have

$$\frac{AN}{NB} = \frac{AC}{BC}, \frac{BL}{LC} = \frac{AB}{AC}, \frac{CM}{MA} = \frac{BC}{AB},$$

from which three equations, by multiplication,

$$\frac{AN}{NB} \cdot \frac{BL}{LC} \cdot \frac{CM}{MA} = \frac{AC}{BC} \cdot \frac{AB}{AC} \cdot \frac{BC}{AB} = 1.$$

Hence AN . BL . CM = NB . LC . MA. Obviously the conditions of our second theorem are again fulfilled, and we have as a conclusion that the bisectors of the angles of a triangle are concurrent.

Incidentally it may be shown that this proof holds for the excenters of a triangle.

(c) *Perpendiculars from the vertices of a triangle to the sides respectively opposite are concurrent:*

Given AD, BE, CF perpendiculars to the respective sides of the given triangle ABC.

We now have three pairs of similar right triangles, since each of the angles A, B, C of our triangle ABC is common to two right triangles. Then from right triangles AFC and AEB, we have

$\frac{AF}{EA} = \frac{AC}{AB}$; from right triangles BDA and BFC, $\frac{BD}{BF} = \frac{AB}{BC}$; and

from right triangles CEB and CDA, $\frac{CE}{CD} = \frac{BC}{AC}$:

Hence from these three equations, by multiplication,

$$\frac{AF}{EA} \cdot \frac{BD}{BF} \cdot \frac{CE}{CD} = 1.$$

Hence AF . BD . CE = EA . FB . DC. We have again fulfilled the conditions of our second theorem. Hence the perpendiculars from the vertices of a triangle on the respective sides opposite are concurrent.

Thus we see that several familiar propositions on concurrent lines have been solved readily and briefly by making them corollaries of one general theorem. So it can be shown with other general theorems many of which can be fully understood and easily handled by the high school student. Their use certainly saves time, saves energy, and gives greater power in the solution of problems, all of which contributes toward increased interest and knowledge in geometry. To this end, every teacher of euclidean geometry should know something of homothetic figures, of the nine-point circle, of the ortho-centric quadrilateral, of harmonic ranges, of Simson's line and of the Theorem of Menelaus, of coaxial circles, and of many other topics well known to the student of advanced euclidean geometry, but not ordinarily mentioned in high school texts.

REMARKS ON GENERALIZATIONS AS MEANS OF MOTIVATION IN ELEMENTARY MATHEMATICS COURSES

By H. A. SIMMONS
Northwestern University

1. *Introduction.* Every good teacher of elementary mathematics spends much time seeking means of getting and holding the interest of his classes. He may present an original practical problem whose solution requires the principle in the text which is to be explained; he may write on the black board a number of partial sentences with blanks interspersed which call for essential facts, and ask the students to fill out these blanks in a short time; he may keep before the class a problem which no member of the class can solve without considerable effort, and give high rating to the first student who solves the problem, etc. It is the purpose of this paper to point out a few *generalizations* which seem to increase interest in classes.

2. *Generalizations.* When one teaches the doctrine of highest common divisors (H. C. D.) and lowest common multiples (L. C. M.),

he always presents the theorem that if A_1 and A_2 are two rational integral functions of a single variable x with H. C. D. = D and L. C. M. =M, then $A_1A_2=MD$. It is a simple mattei to generalize from this theorem to the following one. If $A_1, A_2, \ldots, A_n$ are n integral rational functions of x with H. C. D. = D and L. C. M. = M, then $A_1A_2 \ldots A_n=MD^{n-1}$. Such a generalization, on account of its simplicity, will be clear and satisfying to a class and will emphasize the relation $A_1A_2=MD$.

In *permutations and combinations*, one often finds such a problem as this. In how many ways can 10 books be placed side by side if 3 of them are to be together? A common method of solving this problem is to think of the group of 3 books as one book and to observe that there are 8! possible permutations when the 3 books are together in a given order. Since there are $3!=6$ possible orders for the 3 books, the answer to the problem is $8!\times 3!=241,920$. From this solution, a class can readily see that the number of permutations of m books when n $(<m)$ of them are kept together is $(m-n+1)!\times n!$ Numerous such generalizations can be made while one is teaching *permutations and combinations*, and also *probability*: viz., n pennies can fall in 2^n ways; n dice, in 6^n ways; a man with l coats, m vests, and n pairs of trousers can dress in $l\times m\times n$ ways; the probability of throwing exactly two heads in n throws with one coin, or in one throw with n coins, is $nC_2\div 2^n=n(n-1)\div 2^{n+1}$.

In teaching *logarithms*, after proving that $\log_a x_1x_2=\log_a x_1+\log_a x_2$, one finds a class interested in a proof that $\log_a x_1x_2x_3=\log_a x_1+\log_a x_2+\log_a x_3$. Then, without insisting upon complete induction to the case of n factors, one finds a class glad to tell him what the general theorem is and that the corollary $\log_a x^n = n \log_a x$ must follow. A direct proof of this corollary will then be appreciated by the class.

All of the generalizations suggested above seem to lead students' minds in the direction of *induction*. We believe that if teachers make easy generalizations as often as is convenient in the beginning chapters of algebra, the much dreaded chapter on *mathematical induction* will come to be taught with a good deal of satisfaction. Furthermore such generalizations seem to exert a very wholesome influence on a class, and more specially on its stronger members.

THE COMPLEX NUMBER IN PARTIAL FRACTIONS

By HARRY GWINNER
University of Maryland

In looking over my notes for something interesting for the Mathematics News Letter, the following appealed to me as a suitable choice:

$$\text{Break } \frac{4}{t^4+1} \text{ into partial fractions.}$$

By De Moivre's Theorem (see Bowser's Trigonometry), the denominator which is of the form (x^m+1) may be broken up into the two factors $(x^2-2x\cos\frac{\pi}{4}+1)\ (x^2-2x\cos\frac{3\pi}{4}+1)$.

Therefore; $t^4+1=(t^2-2t\ \cos\frac{\pi}{4}+1)(\ t^2-2t\ \cos\frac{3\pi}{4}+1)$; or $t^4+1=(t^2-t\sqrt{2}+1)\ \ (t^2+t\sqrt{2}+1)$; since $\frac{\pi}{4}=45°$, $\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ $\cos\frac{3}{4}\pi=-\sin\frac{\pi}{4}=-\frac{1}{\sqrt{2}}$.

Assume therefore

$$\text{(a)}\quad \frac{4}{t^4+1}=\frac{At+B}{t^2+t\sqrt{2}+1}+\frac{Ct+D}{t^2-t\sqrt{2}+1}.$$ Clearing of fractions then

$$4=\left\{ t^3 \left| \begin{array}{l} +A \\ +C \end{array} \right. +t^2 \left| \begin{array}{l} -A\sqrt{2} \\ +B \\ +C\sqrt{2} \\ +D \end{array} \right. +t \left| \begin{array}{l} +A \\ -B\sqrt{2} \\ +C \\ +D\sqrt{2} \end{array} \right. +B+D, \right.$$

and equating coefficients of like powers of t, then

$A+C=0\ldots(1), -\sqrt{2}A+B+C\sqrt{2}+D=0\ldots(2)$, $A-B\sqrt{2}+C+D\sqrt{2}=0\ldots(3), B+D=4\ldots(4)$. (1) and (3) give $-B\sqrt{2}+D\sqrt{2}=0\ldots(5)$; $\therefore\ B-D=0$; whence $B=2$, and $D=2$, whence $A=\sqrt{2}$ and $C=-\sqrt{2}$. Substituting these values in (a), then

$$\frac{4}{t^4+1}=\frac{\sqrt{2}t+2}{t^2+t\sqrt{2}+1}+\frac{2-\sqrt{2}t}{t^2-t\sqrt{2}+1}\quad \text{Result.}$$

ON METHODS IN TEACHING GEOMETRY

By C. D. SMITH
Louisiana College

The problem of teaching geometry to high school and college students has been attacked in various ways by teachers of established reputation. It is the purpose of the present note to mention three methods that have been advocated without attempting to settle the question as to which is the better method. It is the hope of the writer that the possibilities and limitations of each of these three methods will be discussed by teachers in future issues of the News Letter.

Although one might not be able to list all desirable criteria for a good method, yet it seems that the following should surely be included in such a list.

1. The method should invite the interest of the student in a variety of ways.
2. Points of difficulty in the course should not be above the mental level of the student.
3. It should be designed to lay the foundation for future progress in geometry.
4. It should be designed to make the best contribution to his education that the time allowed for the course will permit.
5. It should take account of the mental connections that have been previously formed and utilize them when possible in the formation of new connections.

The three methods that I have in mind may be indicated by the following brief descriptions:

A. The teacher conducts the course somewhat as a program of supervised study. Through the aid of suggestions the student is lead to the formation of theorems by experiment. All points in the development are checked and modified by the teacher and other students. A text book is of little use. The result is that the student is lead to the development of a kind of geometry of his own.

B. A text book is placed in the hands of the student. Specific propositions are assigned daily. He is required to establish and defend these propositions against all criticisms of class and teacher. The chief objective seems to be the development of power to demonstrate and defend a geometrical proposition.

C. A text and certain reference books are used. The ability to state accurately the leading facts of geometry is developed. Among such facts are axioms, postulates, theorems, corollaries, descriptions of parallels, perpendiculars, projection, and classes of geometrical figures. The historical background is focused about the origin of the science and the development of the more significant

theorems. Specific references are assigned for written reports. The ability to draw geometrical figures to specifications is cultivated. Necessity of proof is illustrated by special cases. Methods of proof are taught by confining demonstrations to such important propositions as those involving congruent figures, similar figures, areas, volumes, and the like. The amount of demonstration is limited by the time devoted to the course. The validity of all theorems mentioned in the course is illustrated by a numerical case when possible. Frequent reviews are given in the form of objective tests.

In conclusion we may say that it seems fair to assume that the object of any course should be to contribute as much as possible to the general education of the student during the time allotted to the course. In the light of this assumption it is interesting to raise the following questions. Which of these methods would seem to lead to the more desirable contribution? Can you defend your position? What are the merits and demerits of each method? Can you propose a better method and defend your plan? Articles on these points in future issues of the News Letter should be interesting and profitable. Let us hear what high school and college teachers have to say about it.

CORRESPONDENCE

Department of Mathematics,
Louisiana College,
Pineville, La., February 6, 1928.

Professor S. T. Sanders:
Baton Rouge, La.

Dear Professor Sanders:

I think I am due you an explanation if not an apology for not taking a more active part during the last few months in boosting our section of the M. A. of A. I have been occupied almost exclusively with the details connected with the recent convocation of the University of Iowa at which I received my Doctor's degree. That being over I am in a position to offer again my efforts in behalf of the Association. Some time ago I received a letter asking for the subscription price or more to aid in the work of the News Letter. I am inclosing a dollar for this purpose. The News Letter certainly has great possibilities and we should leave nothing undone to make it a real touchstone of mathematical spirit. Seems that it should be one of our most effective methods of enlistment. I hope that it can be put strictly on a subscription basis soon and that a variety of articles on methods of instruction and points of interest in the secondary and collegiate fields will make it a great force among our teachers.

I note that in Vol. 2, No. 3, of the News Letter you are again inviting teachers to offer short articles for future issues of the letter. I hope to aid in this respect in every

way I can. The inclosed article might be of interest to teachers of high school and college geometry. Hope you can use it at some future date. Let me hear from you whenever I can be of service in these parts. With best wishes for your plans in connection with the Jackson meeting, I am,

Cordially yours,
C. D. SMITH.

Mathematics Club, Blue Mountain College,
Blue Mountain, Miss., February 3, 1928.

Prof. S. T. Sanders,
Baton Rouge, La.

My dear Prof. Sanders:

I am sending you a check for one dollar for the Mathematics News Letter. I had hoped to see you at the Nashville meeting and give my subscription to you there.

You have placed the teachers of mathematics under lasting obligations for the work you have done for the Mississippi-Louisiana Section of the Association.

I am planning to attend the meetting at Jackson. Last year sickness prevented my going to Shreveport.

With best wishes, I am,

Yours very truly,
MABEL HUTCHINS.

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Sister M. Michael, 7214 St. Charles Ave., New Orleans, La.
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